

THE B_c -DECAYS $B_c^+ \rightarrow J/\psi \pi^+ \pi^- \pi^+$, $\eta_c \pi^+ \pi^- \pi^+$ Zhi-Gang Wang¹

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Abstract

In this article, we study the three-pion B_c -decays $B_c^+ \rightarrow J/\psi \pi^+ \pi^- \pi^+$, $\eta_c \pi^+ \pi^- \pi^+$ with dominance of the intermediate axial-vector meson $a_1(1260)$ and vector meson $\rho(770)$ in the $\pi^+ \pi^- \pi^+$ and $\pi^+ \pi^-$ invariant mass distributions respectively, and make predictions for the branching fractions and differential decay widths. The ratio between the decays $B_c^+ \rightarrow J/\psi \pi^+ \pi^- \pi^+$ and $B_c^+ \rightarrow J/\psi \pi^+$ is compatible with the experimental data within uncertainties, other predictions can be confronted with the experimental data in the future at the LHCb.

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1 Introduction

In 1998, the CDF collaboration observed the pseudoscalar B_c^\pm mesons through the semileptonic decays $B_c^\pm \rightarrow J/\psi \ell^\pm X$ and $B_c^\pm \rightarrow J/\psi \ell^\pm \bar{\nu}_\ell$ in $p\bar{p}$ collisions at the energy $\sqrt{s} = 1.8$ TeV at the Fermilab Tevatron [1]. The bottom-charm quarkonium states B_c are of special interesting, the ground states and the excited states lying below the BD , BD^* , B^*D , B^*D^* thresholds cannot annihilate into gluons, and therefore are more stable than the corresponding heavy quarkonium states consist of the same flavor, and would have widths less than a hundred KeV [2]. The semileptonic decays $B_c^\pm \rightarrow J/\psi \ell^\pm \bar{\nu}_\ell$, $B_c^+ \rightarrow J/\psi e^+ \bar{\nu}_e$ were used to measure the B_c lifetime [3, 4], which is about a third as long as that of the B and B_s mesons as both the b and c quarks decay weakly. In 2007, the CDF collaboration observed the B_c^\pm mesons with a significance exceeds 8σ through the decays $B_c^\pm \rightarrow J/\psi \pi^\pm$ in $p\bar{p}$ collisions at the energy $\sqrt{s} = 1.96$ TeV, and obtained the value $m_{B_c} = (6275.6 \pm 2.9 \pm 2.5)$ MeV [5]. In 2008, the D0 collaboration reconstructed the decay modes $B_c^\pm \rightarrow J/\psi \pi^\pm$ and observed the B_c^\pm mesons with a significance larger than 5σ , and obtained the value $m_{B_c} = (6300 \pm 14 \pm 5)$ MeV [6]. Now the average value is $m_{B_c} = (6.277 \pm 0.006)$ GeV from the Particle Data Group [7]. Recently, the LHCb collaboration observed the decay $B_c^+ \rightarrow J/\psi \pi^+ \pi^- \pi^+$ for the first time using 0.8 fb^{-1} of the pp collisions at $\sqrt{s} = 7$ TeV, the measured ratio of branching fractions is

$$\frac{Br(B_c^+ \rightarrow J/\psi \pi^+ \pi^- \pi^+)}{Br(B_c^+ \rightarrow J/\psi \pi^+)} = 2.41 \pm 0.30 \pm 0.33, \quad (1)$$

where the first uncertainty is statistical and the second is systematic [8].

The hadronic decays $B_c^+ \rightarrow J/\psi \pi^+ \pi^- \pi^+$, $J/\psi \pi^+$ take place through the weak b -quark decays $b \rightarrow cW^* \rightarrow c\bar{u}d$, which are analogous with the τ -lepton decays $\tau \rightarrow \nu_\tau W^* \rightarrow \nu_\tau \bar{u}d$. We can use the existing experimental data on the τ -lepton decays to obtain reliable prediction for the $B_c^+ \rightarrow J/\psi \pi^+ \pi^- \pi^+$ branching fraction with the method of spectral functions,

$$\int d\Phi(W^* \rightarrow \pi^+ \pi^- \pi^+) \epsilon_\mu \epsilon_\nu^* = (q_\mu q_\nu - q^2 g_{\mu\nu}) \rho_T(q^2) + q_\mu q_\nu \rho_L(q^2), \quad (2)$$

where the $d\Phi(W^* \rightarrow \pi^+ \pi^- \pi^+)$ is the Lorentz-invariant three-body phase factor, the ϵ_μ is the effective polarization vector of the virtual W -boson, the spectral functions $\rho_T(q^2)$ and $\rho_L(q^2)$ are universal and can be determined by theoretical and experimental analysis of the $\tau \rightarrow \nu_\tau \pi^+ \pi^- \pi^+$ decays [9, 10, 11, 12]. The spectral function $\rho_L(q^2)$ is negligible according to conservation of vector

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current and partial conservation of axial-vector current. The explicit expression of the spectral function $\rho_T(q^2)$ can be fitted to the experimental data or calculated by some phenomenological models, the spectral function $\rho_T(q^2)$ is always saturated by exchange of the intermediate axial-vector meson $a_1(1260)$ with an special ansatz for the vertexes $a_1\rho\pi$ and $\rho\pi\pi$ based on some phenomenological quark models [13, 14, 15, 16]. In this article, we intend to study the decays $B_c^+ \rightarrow J/\psi \pi^+ \pi^- \pi^+$, $\eta_c \pi^+ \pi^- \pi^+$, $J/\psi \pi^+$, $\eta_c \pi^+$ with the phenomenological Lagrangians, and calculate the Feynman diagrams directly. The decays $B_c^+ \rightarrow \eta_c \pi^+ \pi^- \pi^+$, $\eta_c \pi^+$ have not been observed yet, but they are expected to be observed in the future at the Large Hadron Collider (LHC). The B_c decays will be studied and their branching fractions will be determined in the LHCb experiments.

In Ref.[17], Lichard and Juran perform detailed analysis of the vertex $a_1\rho\pi$ with the following Lagrangian,

$$\mathcal{L} = \frac{g_{a\rho\pi}}{\sqrt{2}} \{ \cos\theta [(\partial^\mu \rho^{0\nu} - \partial^\nu \rho^{0\mu}) a_\mu^- \partial_\nu \pi^+] - \sin\theta [(\partial^\mu \rho^{0\nu} - \partial^\nu \rho^{0\mu}) \partial_\mu a_\nu^- \pi^+] + \dots \}, \quad (3)$$

considering the decays $a_1(1260) \rightarrow \rho\pi$, where the momenta of the ρ and π mesons in center-of-mass frame of the initial $a_1(1260)$ meson are about 0.37 GeV. Such a Lagrangian maybe lead to amplified amplitude for the decay $B_c^+ \rightarrow J/\psi \pi^+ \pi^- \pi^+$ as the sub-amplitude $a_1^+(1260) \rightarrow \rho\pi^+$ can be accounted as $g_{a\rho\pi} l_\rho \cdot k_\pi$ or $g_{a\rho\pi} l_\rho \cdot q_a$, where the momenta q_a , l_ρ and k_π are large, we have to introduce form-factors to parameterize the off-shell effects. In the decays $\tau \rightarrow a_1(1260) \nu_\tau$ and $B_c^+ \rightarrow J/\psi a_1^+(1260)$, the momenta of the $a_1(1260)$ meson in center-of-mass frame of the initial particles are about 0.45 GeV and 2.16 GeV, respectively. On the other hand, we know that the decays $a_1(1260) \rightarrow \rho\pi$ are S -wave dominated [7], we prefer the simple Lagrangian,

$$\mathcal{L}_{a\rho\pi} = g_{a\rho\pi} a_1^\mu \rho_\mu \pi, \quad (4)$$

in this article. Furthermore, we use the Lagrangian,

$$\mathcal{L}_{\rho\pi\pi} = -ig_{\rho\pi\pi} \rho^{0\mu} \pi^- (\vec{\partial}_\mu - \overleftarrow{\partial}_\mu) \pi^+, \quad (5)$$

to study the vertex $\rho\pi\pi$ [18].

The article is arranged as follows: we derive the decay widths of the processes $B_c^+ \rightarrow J/\psi \pi^+ \pi^- \pi^+$, $\eta_c \pi^+ \pi^- \pi^+$, $J/\psi \pi^+$, $\eta_c \pi^+$ in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusions.

2 Decay widths of the processes $B_c^+ \rightarrow J/\psi \pi^+ \pi^- \pi^+$, $\eta_c \pi^+ \pi^- \pi^+$

The hadronic decays $B_c^+ \rightarrow J/\psi \pi^+$, $\eta_c \pi^+$, $J/\psi \pi^+ \pi^- \pi^+$ and $\eta_c \pi^+ \pi^- \pi^+$ can be described by the effective Hamiltonian,

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* C_1(\mu) \bar{c} \gamma_\alpha (1 - \gamma_5) b \bar{d} \gamma^\alpha (1 - \gamma_5) u + h.c., \quad (6)$$

where the V_{cb} , V_{ud} are the CKM matrix elements, the G_F is the Fermi constant, and the $C_1(\mu)$ is the Wilson coefficient defined at an special energy scale, $C_1(m_b) = 1.14$ [19]. In the following, we write down the definitions for the weak form-factors $F_1(q^2)$, $F_0(q^2)$, $A_1(q^2)$, $A_2(q^2)$, $A_3(q^2)$, $A_0(q^2)$ and $V(q^2)$ for the current $J_\mu = \bar{c} \gamma_\mu (1 - \gamma_5) b$ sandwiched between the B_c and the η_c , J/ψ

[20],

$$\begin{aligned}\langle \eta_c(p) | J_\mu(0) | B_c(P) \rangle &= \left[(P+p)_\mu - \left(m_{B_c}^2 - m_{J/\psi}^2 \right) \frac{q_\mu}{q^2} \right] F_1(q^2) + \left(m_{B_c}^2 - m_{J/\psi}^2 \right) \frac{q_\mu}{q^2} F_0(q^2), \\ &= (P+p)_\mu F_+(q^2) + q_\mu F_-(q^2),\end{aligned}\quad (7)$$

$$\begin{aligned}\langle J/\psi(p) | J_\mu(0) | B_c(P) \rangle &= i \left\{ -\epsilon_\mu^* (m_{B_c} + m_{J/\psi}) A_1(q^2) + \epsilon^* \cdot P (P+p)_\mu \frac{A_2(q^2)}{m_{B_c} + m_{J/\psi}} + \right. \\ &\quad \left. 2m_{J/\psi} \epsilon^* \cdot P \frac{q_\mu}{q^2} [A_3(q^2) - A_0(q^2)] - i \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} q^\alpha (P+p)^\beta \frac{V(q^2)}{m_{B_c} + m_{J/\psi}} \right\}, \\ &= i \left\{ -\epsilon_\mu^* (m_{B_c} + m_{J/\psi}) A_1(q^2) + \epsilon^* \cdot P (P+p)_\mu \frac{A_+(q^2)}{m_{B_c} + m_{J/\psi}} + \right. \\ &\quad \left. \epsilon^* \cdot P q_\mu \frac{A_-(q^2)}{m_{B_c} + m_{J/\psi}} - i \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} q^\alpha (P+p)^\beta \frac{V(q^2)}{m_{B_c} + m_{J/\psi}} \right\},\end{aligned}\quad (8)$$

where

$$\begin{aligned}F_+(q^2) &= F_1(q^2), \\ F_-(q^2) &= \frac{[F_0(q^2) - F_1(q^2)] (m_{B_c}^2 - m_{J/\psi}^2)}{q^2}, \\ A_3(q^2) &= \frac{m_{B_c} + m_{J/\psi}}{2m_{J/\psi}} A_1(q^2) - \frac{m_{B_c} - m_{J/\psi}}{2m_{J/\psi}} A_2(q^2), \\ A_+(q^2) &= A_2(q^2), \\ A_-(q^2) &= \frac{2m_{J/\psi}(m_{B_c} + m_{J/\psi})}{q^2} [A_3(q^2) - A_0(q^2)],\end{aligned}\quad (9)$$

and $V_0(0) = V_3(0)$, and the ϵ_μ is the polarization vector of the vector meson J/ψ .

There have been several approaches to calculate those weak form-factors, such as the QCD sum rules [19, 21, 22], the light-cone QCD sum rules [23], the relativistic quark models [24, 25, 26, 27], the light-front quark models [28, 29], the non-relativistic quark models [30, 31], perturbative QCD [32], etc. In this article, we take the typical values from the QCD sum rules [19], the relativistic quark models [24, 25] and the light-front quark models [28], and refer them as QCDSR, RQM1, RQM2 and LFQM, respectively. In calculations, we use the following definition for the decay constant f_a of the axial-vector meson $a_1(1260)$,

$$\langle 0 | \bar{u}(0) \gamma_\mu \gamma_5 d(0) | a_1(1260) \rangle = f_a m_a \varepsilon_\mu, \quad (10)$$

where the ε_μ is the polarization vector.

We take into account the effective Hamiltonian \mathcal{H}_{eff} , the weak form-factors and the Lagrangians $\mathcal{L}_{a\rho\pi}$, $\mathcal{L}_{\rho\pi\pi}$ to obtain the amplitudes $T_{J/\psi\pi^+\pi^-\pi^+}$, $T_{\eta_c\pi^+\pi^-\pi^+}$ of the processes $B_c^+ \rightarrow J/\psi \pi^+ \pi^- \pi^+$, $\eta_c \pi^+ \pi^- \pi^+$,

$$\begin{aligned}T_j &= \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* C_1(m_b) f_a m_a \tilde{T}_j, \\ \tilde{T}_j &= \tilde{T}_j^1 + \tilde{T}_j^2,\end{aligned}\quad (11)$$

where $j = J/\psi \pi^+ \pi^- \pi^+$, $\eta_c \pi^+ \pi^- \pi^+$,

$$\begin{aligned} \tilde{T}_{J/\psi \pi^+ \pi^- \pi^+}^1 &= i \left\{ -\epsilon_\mu^* (m_{B_c} + m_{J/\psi}) A_1(q^2) + \epsilon^* \cdot P (P + p)_\mu \frac{A_+(q^2)}{m_{B_c} + m_{J/\psi}} + \epsilon^* \cdot P q_\mu \frac{A_-(q^2)}{m_{B_c} + m_{J/\psi}} \right. \\ &\quad \left. - i \epsilon_{\mu\lambda\tau\sigma} \epsilon^{*\lambda} q^\tau (P + p)^\sigma \frac{V(q^2)}{m_{B_c} + m_{J/\psi}} \right\} \frac{i}{q^2 - m_a^2 + i\sqrt{q^2} \Gamma_a(q^2)} \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) i g_{a\rho\pi} \\ &\quad \frac{i}{l^2 - m_\rho^2 + i\sqrt{l^2} \Gamma_\rho(l^2)} \left(-g_{\nu\alpha} + \frac{l_\nu l_\alpha}{l^2} \right) i g_{\rho\pi\pi} (t - r)^\alpha, \end{aligned} \quad (12)$$

$$\tilde{T}_{J/\psi \pi^+ \pi^- \pi^+}^2 = \tilde{T}_{J/\psi \pi^+ \pi^- \pi^+}^1 (\pi^+(r) \leftrightarrow \pi^+(k)), \quad (13)$$

$$\begin{aligned} \tilde{T}_{\eta_c \pi^+ \pi^- \pi^+}^1 &= \{ (P + p)_\mu F_+(q^2) + q_\mu F_-(q^2) \} \frac{i}{q^2 - m_a^2 + i\sqrt{q^2} \Gamma_a(q^2)} \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) i g_{a\rho\pi} \\ &\quad \frac{i}{l^2 - m_\rho^2 + i\sqrt{l^2} \Gamma_\rho(l^2)} \left(-g_{\nu\alpha} + \frac{l_\nu l_\alpha}{l^2} \right) i g_{\rho\pi\pi} (t - r)^\alpha, \end{aligned} \quad (14)$$

$$\tilde{T}_{\eta_c \pi^+ \pi^- \pi^+}^2 = \tilde{T}_{\eta_c \pi^+ \pi^- \pi^+}^1 (\pi^+(r) \leftrightarrow \pi^+(k)), \quad (15)$$

$$\begin{aligned} \Gamma_a(q^2) &= \Gamma_a \frac{m_a^2}{q^2} \left(\frac{q^2 - 9m_\pi^2}{m_a^2 - 9m_\pi^2} \right)^{\frac{3}{2}}, \\ \Gamma_\rho(l^2) &= \Gamma_\rho \frac{m_\rho^2}{l^2} \left(\frac{l^2 - 4m_\pi^2}{m_\rho^2 - 4m_\pi^2} \right)^{\frac{3}{2}}, \end{aligned} \quad (16)$$

then obtain the differential decay widths

$$d\Gamma_j = \frac{|T_j|^2}{4m_{B_c}} \frac{dq^2}{2\pi} \frac{dl^2}{2\pi} d\Phi(P \rightarrow q, p) d\Phi(q \rightarrow l, k) d\Phi(l \rightarrow r, t), \quad (17)$$

where the $d\Phi(P \rightarrow q, p)$, $d\Phi(q \rightarrow l, k)$, $d\Phi(l \rightarrow r, t)$ are the two-body phase factors defined analogously, for example,

$$d\Phi(P \rightarrow q, p) = (2\pi)^4 \delta^4(P - q - p) \frac{d^3\vec{p}}{(2\pi)^3 2p_0} \frac{d^3\vec{q}}{(2\pi)^3 2q_0}. \quad (18)$$

The decay widths of the processes $B_c^+ \rightarrow J/\psi \pi^+$, $\eta_c \pi^+$ can be calculated straightforward using the effective Hamiltonian \mathcal{H}_{eff} and the weak form-factors, the explicit expressions are neglected for simplicity.

3 Numerical results and discussions

The input parameters are taken as $G_F = 1.166364 \times 10^{-5} \text{ GeV}^{-2}$, $V_{ud} = 0.97425$, $V_{cb} = 40.6 \times 10^{-3}$, $m_\pi = 139.57 \text{ MeV}$, $m_\rho = 775.49 \text{ MeV}$, $\Gamma_\rho = 146.2 \text{ MeV}$, $m_{B_c} = 6.277 \text{ GeV}$, $\tau_{B_c} = 0.45 \times 10^{-12} \text{ s}$ from the Particle Data Group [7], $m_a = 1255 \text{ MeV}$, $\Gamma_a = 367 \text{ MeV}$ from the COMPASS collaboration [33], $g_{\rho\pi\pi} = 6.05$ from the decay $\rho \rightarrow \pi\pi$ [18], $g_{a\rho\pi} = 3.37$ from the decay $a_1(1260) \rightarrow \rho\pi$ [7], $f_a = 0.24 \text{ GeV}$ from the QCD sum rules [34].

We obtain the branching fractions of the decays $B_c^+ \rightarrow J/\psi \pi^+$, $\eta_c \pi^+$, $J/\psi \pi^+ \pi^- \pi^+$ and $\eta_c \pi^+ \pi^- \pi^+$ with the typical values of the weak form-factors from the QCD sum rules [19], the relativistic quark models [24, 25], and the light-front quark models [28]. The form-factors in Refs.[19, 24, 25] are fitted to an single pole form,

$$f(q^2) = \frac{f(0)}{1 - q^2/m_{f_{it}}^2}, \quad (19)$$

	$A_1(0)$ [m_{fit}]	$A_+(0)$ [m_{fit}]	$A_-(0)$ [m_{fit}]	$V(0)$ [m_{fit}]	$F_+(0)$ [m_{fit}]	$F_-(0)$ [m_{fit}]
QCDSR [19]	0.63 [4.5]	0.69 [4.5]	-1.12 [4.5]	1.03 [4.5]	0.66 [4.5]	-0.36 [4.5]
RQM1 [24]	0.56 [5.45]	0.54 [4.76]	-0.95 [4.68]	0.83 [4.72]	0.61 [4.84]	-0.32 [4.80]
	$A_1(0)$ [m_{fit}]	$A_2(0)$ [m_{fit}]	$A_0(0)$ [m_{fit}]	$V(0)$ [m_{fit}]	$F_1(0)$ [m_{fit}]	$F_0(0)$ [m_{fit}]
RQM2 [25]	0.50 [4.84]	0.73 [4.72]	0.40 [4.04]	0.49 [3.99]	0.47 [4.41]	0.47 [4.72]
	$A_1(0)$ [c_1/c_2]	$A_2(0)$ [c_1/c_2]	$A_0(0)$ [c_1/c_2]	$V(0)$ [c_1/c_2]	$F_1(0)$ [c_1/c_2]	$F_0(0)$ [c_1/c_2]
LFQM [28]	0.50 [1.73/0.33]	0.44 [2.22/0.45]	0.53 [2.39/0.50]	0.74 [2.46/0.56]	0.61 [1.99/0.44]	0.61 [1.18/0.17]

Table 1: The parameters for the weak form-factors, where the unit of the m_{fit} is GeV.

while the form-factors in Ref.[28] are fitted to an exponential form,

$$f(q^2) = f(0) \exp(c_1 q^2 + c_2 q^4), \quad (20)$$

where the $f(q^2)$ denote the weak form-factors, the m_{fit} , c_1 , c_2 are fitted parameters. The numerical values are presented explicitly in Table 1.

The numerical values of the branching fractions are shown in Table 2, from the table, we can see that the branching fractions vary in a large range according to the values of the weak form-factors from different theoretical approaches, it is difficult to determine which one is superior to others. In Table 2, we also present the predictions from the Berezhnuy-Likhoded-Luchinsky (BLL) model for comparison, where the spectral function

$$\rho_T(q^2) = 5.86 \times 10^{-5} \left(1 - \frac{9m_\pi^2}{q^2}\right)^4 \frac{1 + 190q^2}{[(q^2 - 1.06)^2 + 0.48]^2} \quad (21)$$

determined from the $\tau \rightarrow \nu_\tau \pi^+ \pi^- \pi^+$ decays is used [9, 10, 11]. The present values are slightly different from that of Ref.[9], as we have taken slightly different input parameters.

The ratios among the branching fractions are shown explicitly in Table 3, from the table, we can see that

$$\frac{Br(B_c^+ \rightarrow J/\psi \pi^+ \pi^- \pi^+)}{Br(B_c^+ \rightarrow J/\psi \pi^+)} = 2.17, 1.87, 2.28, 1.88, \quad (22)$$

are all compatible with the experimental data $2.41 \pm 0.30 \pm 0.33$ within uncertainties [8], while the ratios based on the weak form-factors from the QCD sum rules in Ref.[19] and the relativistic quark models in Ref.[25] are better. All those predictions can be confronted with the experimental data in the future at the LHCb collaboration.

In Fig.1, we plot the differential decay widths of the B_c mesons $d\Gamma(B_c^+ \rightarrow J/\psi \pi^+ \pi^- \pi^+)/dq^2$, $d\Gamma(B_c^+ \rightarrow \eta_c \pi^+ \pi^- \pi^+)/dq^2$, $d\Gamma(B_c^+ \rightarrow J/\psi \pi^+ \pi^- \pi^+)/dl^2$ and $d\Gamma(B_c^+ \rightarrow \eta_c \pi^+ \pi^- \pi^+)/dl^2$ with variations of the squared momenta q^2 ($M^2(\pi^+ \pi^- \pi^+)$) and l^2 ($M^2(\pi^+ \pi^-)$). By measuring the momenta dependence of the differential decay widths, we can test dominance of the intermediate axial-vector meson $a_1(1260)$ and vector meson $\rho(770)$ in the invariant $\pi^+ \pi^- \pi^+$ and $\pi^+ \pi^-$ mass distributions, respectively. It is difficult to distinguish the two π^+ mesons in the final states, so we should not be serious for taking the squared virtual momentum l^2 as the invariant mass distribution

	QCDSR [19]	RQM1 [24]	RQM2 [25]	LFQM [28]
$B_c^+ \rightarrow J/\psi \pi^+$	1.222	1.106	0.544	0.956
$B_c^+ \rightarrow \eta_c \pi^+$	1.592	1.360	0.879	1.360
$B_c^+ \rightarrow J/\psi \pi^+ \pi^- \pi^+$	2.655 [3.475]	2.072 [2.758]	1.242 [1.607]	1.797 [2.390]
$B_c^+ \rightarrow \eta_c \pi^+ \pi^- \pi^+$	1.854	1.544	0.947	1.578

Table 2: The branching fractions of the B_c decays, where the unit is 10^{-3} , and the references denote the hadronic form-factors from that articles are used. The values in the bracket denote the predictions from the BLL model, where the spectral function $\rho_T(q^2)$ determined from the $\tau \rightarrow \nu_\tau \pi^+ \pi^- \pi^+$ decays is used.

	QCDSR [19]	RQM1 [24]	RQM2 [25]	LFQM [28]
$\frac{Br(B_c^+ \rightarrow J/\psi \pi^+ \pi^- \pi^+)}{Br(B_c^+ \rightarrow J/\psi \pi^+)}$	2.17	1.87	2.28	1.88
$\frac{Br(B_c^+ \rightarrow \eta_c \pi^+ \pi^- \pi^+)}{Br(B_c^+ \rightarrow J/\psi \pi^+)}$	1.52	1.40	1.74	1.65
$\frac{Br(B_c^+ \rightarrow \eta_c \pi^+)}{Br(B_c^+ \rightarrow J/\psi \pi^+)}$	1.30	1.23	1.62	1.42
$\frac{Br(B_c^+ \rightarrow \eta_c \pi^+ \pi^- \pi^+)}{Br(B_c^+ \rightarrow \eta_c \pi^+)}$	1.16	1.14	1.08	1.16

Table 3: The ratios among the branching fractions of the B_c decays, where the references denote the hadronic form-factors from that articles are used.

$M^2(\pi^+ \pi^-)$. However, from Fig.2 we can see that the approximation $l^2 = M^2(\pi^+ \pi^-)$ works rather well.

In Fig.2, we plot the $\pi^+ \pi^- \pi^+$ and $\pi^+ \pi^-$ invariant mass distributions in the decays $B_c \rightarrow J/\psi \pi^+ \pi^- \pi^+$ compared with the experimental data and the predictions of the BLL model. With suitable normalization, the weak form-factors from the QCD sum rules [19], the relativistic quark models [24, 25], and the light-front quark models [28] lead to almost the same line-shapes for the invariant mass distributions, although they correspond to quite different decay widths. From the figure, we can see that the present work and the BLL model both describe the experimental data on the invariant mass distributions $M(\pi^+ \pi^- \pi^+)$ well, while the present prediction is better.

4 Conclusion

In this article, we study the three-pion B_c -decays $B_c^+ \rightarrow J/\psi \pi^+ \pi^- \pi^+$, $\eta_c \pi^+ \pi^- \pi^+$ assuming dominance of the intermediate axial-vector meson $a_1(1260)$ and vector meson $\rho(770)$ in the invariant $\pi^+ \pi^- \pi^+$ and $\pi^+ \pi^-$ mass distributions respectively, and make predictions for the branching fractions and differential decay widths. The ratios between the decays $B_c^+ \rightarrow J/\psi \pi^+ \pi^- \pi^+$ and $B_c^+ \rightarrow J/\psi \pi^+$ based on the form-factors from different theoretical approaches are compatible with the experimental data within uncertainties. The decays $B_c^+ \rightarrow \eta_c \pi^+ \pi^- \pi^+$, $\eta_c \pi^+$ have not been observed yet, the predictions can be confronted with the experimental data in the future at the LHCb.

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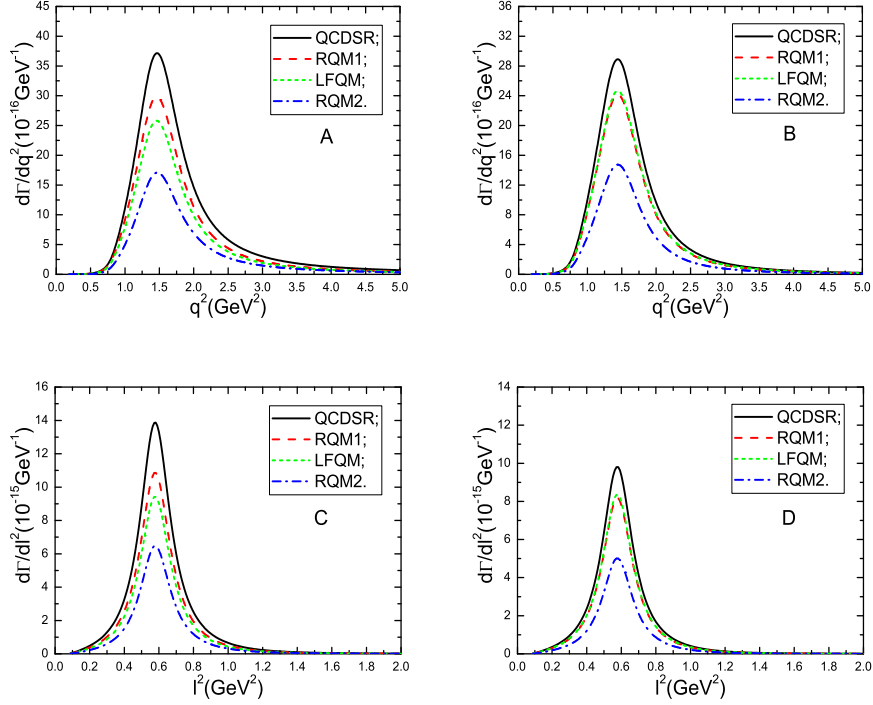


Figure 1: The differential decay widths of the B_c meson, where the A , B , C and D denote the differential decay widths $d\Gamma(B_c^+ \rightarrow J/\psi \pi^+ \pi^- \pi^+)/dq^2$, $d\Gamma(B_c^+ \rightarrow \eta_c \pi^+ \pi^- \pi^+)/dq^2$, $d\Gamma(B_c^+ \rightarrow J/\psi \pi^+ \pi^- \pi^+)/dl^2$ and $d\Gamma(B_c^+ \rightarrow \eta_c \pi^+ \pi^- \pi^+)/dl^2$, respectively.

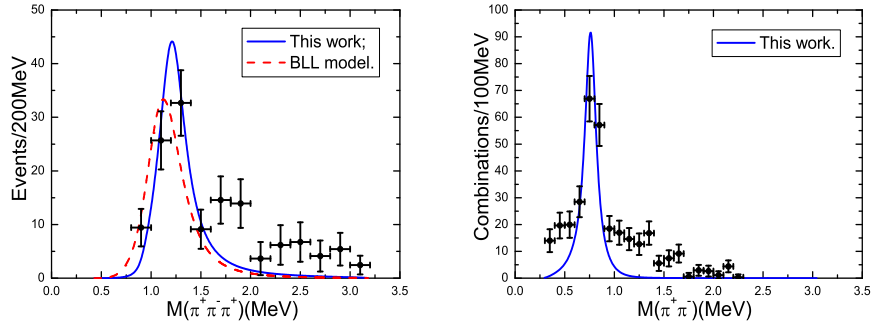


Figure 2: The invariant mass distributions of the $\pi^+ \pi^- \pi^+$ and $\pi^+ \pi^-$ compared with the experimental data, where we have taken the approximation $l^2 = M^2(\pi^+ \pi^-)$.

References

- [1] F. Abe et al, Phys. Rev. Lett. **81** (1998) 2432; F. Abe et al, Phys. Rev. **D58** (1998) 112004.
- [2] S. Godfrey and N. Isgur, Phys. Rev. **D32** (1985) 189; S. Godfrey, Phys. Rev. **D70** (2004) 054017.
- [3] A. Abulencia et al, Phys. Rev. Lett. **97** (2006) 012002.
- [4] V. Abazov et al, Phys. Rev. Lett. **102** (2009) 092001.
- [5] T. Aaltonen et al, Phys. Rev. Lett. **100** (2008) 182002.
- [6] V. M. Abazov et al, Phys. Rev. Lett. **101** (2008) 012001.
- [7] K. Nakamura et al, J. Phys. **G37** (2010) 075021.
- [8] R. Aaij et al, Phys. Rev. Lett. **108** (2012) 251802.
- [9] A. K. Likhoded and A. V. Luchinsky, Phys. Rev. **D81** (2010) 014015.
- [10] A. V. Berezhnoy, A. K. Likhoded and A. V. Luchinsky, arXiv:1104.0808.
- [11] A. V. Berezhnoy, A. K. Likhoded and A. V. Luchinsky, PoS QFTHEP2011 (2011) 076.
- [12] A. Rakitin and S. Koshkarev, Phys. Rev. **D81** (2010) 014005.
- [13] J. H. Kuhn and A. Santamaria, Z. Phys. **C48** (1990) 445.
- [14] B. A. Li, Phys. Rev. **D55** (1997) 1436.
- [15] Y. P. Ivanov, A. A. Osipov and M. K. Volkov, Z. Phys. **C49** (1991) 563.
- [16] J. H. Kuhn and E. Mirkes, Phys. Lett. **B286** (1992) 381.
- [17] P. Lichard and J. Juran, Phys. Rev. **D76** (2007) 094030.
- [18] H. Y. Cheng, C. K. Chua and A. Soni, Phys. Rev. **D71** (2005) 014030.
- [19] V. V. Kiselev, hep-ph/0211021.
- [20] M. Wirbel, B. Stech and M. Bauer, Z. Phys. **C29** (1985) 637.
- [21] V. V. Kiselev, A. K. Likhoded and A. I. Onishchenko, Nucl. Phys. **B569** (2000) 473.
- [22] P. Colangelo, G. Nardulli and N. Paver, Z. Phys. **C57** (1993) 43.
- [23] T. Huang and F. Zuo, Eur. Phys. J. **C51** (2007) 833.
- [24] M. A. Ivanov, J. G. Korner and P. Santorelli, Phys. Rev. **D71** (2005) 094006.
- [25] D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. **D68** (2003) 094020.
- [26] M. A. Ivanov, J. G. Korner and P. Santorelli, Phys. Rev. **D63** (2001) 074010.
- [27] M. A. Nobes and R. M. Woloshyn, J. Phys. **G26** (2000) 1079.
- [28] W. Wang, Y. L. Shen and C. D. Lu, Phys. Rev. **D79** (2009) 054012.
- [29] H. M. Choi and C. R. Ji, Phys. Rev. **D80** (2009) 054016.
- [30] E. Hernandez, J. Nieves and J.M. Verde-Velasco, Phys. Rev. **D74** (2006) 074008.

- [31] R. Dhir and R. C. Verma, Phys. Rev. **D79** (2009) 034004.
- [32] J. F. Sun, D. S. Du and Y. L. Yang, Eur. Phys. J. **C60** (2009) 107.
- [33] M. G. Alekseev et al, Phys. Rev. Lett. **104** (2010) 241803.
- [34] K. C. Yang, Nucl. Phys. **B776** (2007) 187.